UNIT- 2

CONDUCTORS AND DIELECTRICS

INTRODUCTION:

CURRENT AND CURRENT DENSITY

CURRENT: Current is defined as the rate of flow of Charge.

UNIT OF CHARGE: Amperes. (A)

CURRENT DENSITY: It is defined as the current passing through the unit surface area, when the surface is held normal to the direction of the current.

UNIT OF CURRENT DENSITY: Amperes per Square metres (A/m^2)

CURRENT

1. Drift current
2. Displacement or convection current

DRIFT CURRENT: Exists in conductors, due to the drifting of electrons, under the influence of the applied voltage.

DISPLACEMENT CURRENT: Exists in dielectrics, due to the flow of charges under the influence of electric field ($\bar{E}$)

Eg: current flowing through capacitor.

ELECTRIC CURRENT

Flow of charge Per unit time is called an electric current. $I = \frac{dQ}{dt}$.

UNIT is Ampere or Coulomb/Second (C/s)

CONVENTIONAL CURRENT → Current flow from -ve terminal to the +ve terminal.

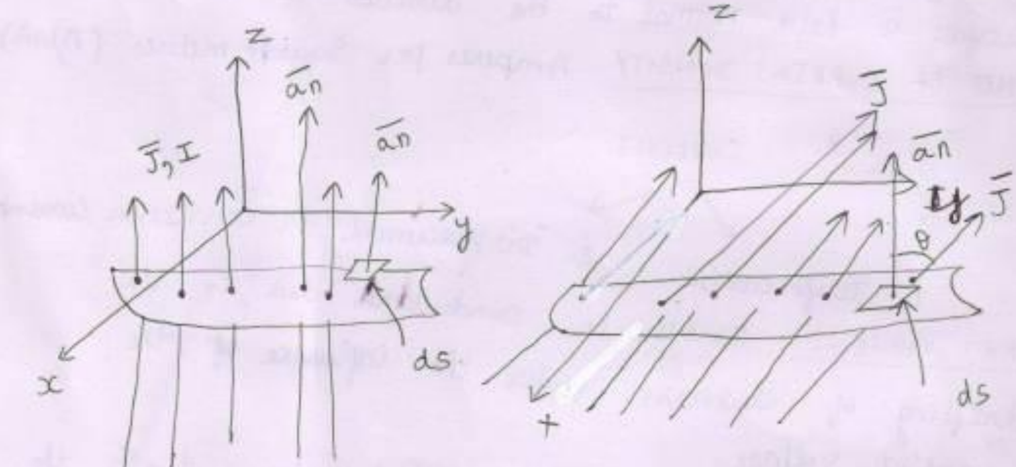
ELECTRON FLOW CURRENT → Current flow from +ve terminal to the -ve terminal

RELATION BETWEEN I and $\bar{J}$

→ The current density is a vector quantity associated with current I

→ It is denoted as $\bar{J}$.

$\bar{J}$ & $d\bar{s}$ are normal $\bar{J}$ & $d\bar{s}$ are not at right angles.



$d\bar{s} = ds\,\bar{a}_n \longrightarrow (1)$

$ds \rightarrow$ Incremental surface area;

$\bar{a}_n \rightarrow$ unit vector

Current Density vector $\bar{J} = J\,\bar{a}_n \rightarrow (2)$

Differential current $dI = \bar{J}\cdot d\bar{s}$ (dot product) $\longrightarrow (3)$

when $\bar{J}$ & $d\bar{s}$ are at right angles ($\theta = 90°$)

then $dI = J\,\bar{a}_n \cdot ds\,\bar{a}_n = J\,ds$ $(\bar{a}_n\cdot\bar{a}_n = 1)$

$dI = J\,ds$

$\int dI = \int_s J\,ds$; $\boxed{I = \int_s J\cdot ds} \rightarrow (4)$

where

$J \rightarrow$ current density (A/m^2)

$d\bar{S} \rightarrow$ differential surface area (m^2).

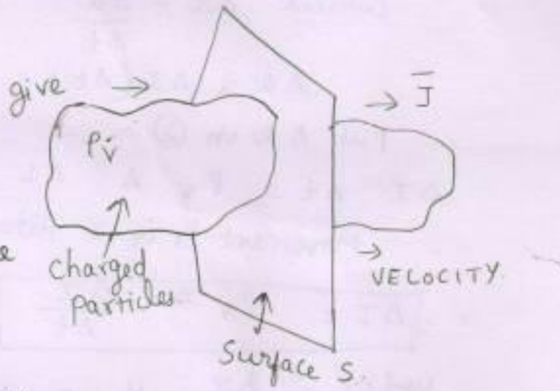
$I \rightarrow$ current (Amperes (A))

$$I = \int_S \bar{J} \cdot d\bar{S} \quad \text{(Dot product)}$$

Incremental current dI is the dot product of $\bar{J}$ and $d\bar{S}$ over the surface S.

RELATION BETWEEN $\bar{J}$ and ρ_v

→ set q charged particles give rise to a charge density ρ_v in a volume V

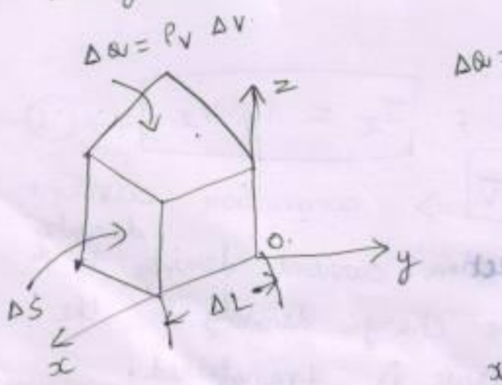


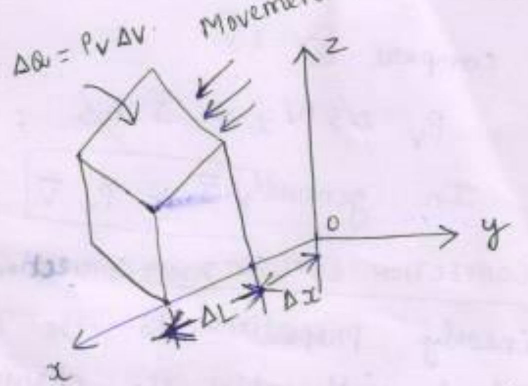
→ $\bar{J}$ (current density) can be related to the velocity with which ρ_v

(ie) charged particles in volume V crosses the surface S at a point

→ velocity with which the Charge is getting transferred.

Movement in x direction





Incremental charge moving in x - direction.

$$\Delta Q = \rho_v \Delta V \rightarrow ①$$

where

$\Delta V \rightarrow$ differential volume

$\Delta Q \rightarrow$ differential charge

$\Delta V = \Delta S\, \Delta L$; $\Delta S \rightarrow$ Incremental surface, $\Delta L \rightarrow$ Incremental Length

put ΔV in ①

$$\boxed{\Delta Q = \rho_v\, \Delta S\, \Delta L} \rightarrow ②$$

Current $\Delta I = \frac{\Delta Q}{\Delta t}$

$$\Delta Q = \Delta I(\Delta t)$$

put ΔQ in ②

$\Delta I\, \Delta t = \rho_v\, \Delta S\, \Delta L$; $\boxed{\Delta I = \frac{\rho_v\, \Delta S\, \Delta L}{\Delta t}} \rightarrow ③$

Movement is in x direction, $\Delta L = \Delta x$.

$$\boxed{\Delta I = \rho_v\, \Delta S\, \frac{\Delta x}{\Delta t}} \rightarrow ④$$

where, $\frac{\Delta x}{\Delta t} = v_x =$ velocity in x direction.

$$\boxed{\Delta I = \rho_v\, \Delta S\, v_x} \rightarrow ⑤$$

when $\bar{J}$ & ΔS are normal, $\boxed{\Delta I = \bar{J}\, \Delta S} \rightarrow ⑥$

compare ⑤ & ⑥.

$\rho_v\, \cancel{\Delta S}\, v_x = \bar{J}\, \cancel{\Delta S}$; $\boxed{J_x = \rho_v\, v_x} \rightarrow ⑦$

In general $\boxed{\bar{J} = \rho_v \bar{v}} \Rightarrow$ convection current density

CONVECTION CURRENT DENSITY Convection current density is linearly proportional to the charge density & the velocity with which the charge is transferred.

CONTINUITY EQUATION OF CURRENT:

→ It is based on Principle of conservation of charge.

→ The charges can neither be created nor be destroyed.

→ consider a closed surface 'S' with current density $\bar{J}$,

→ Total current I crossing the surface S,

$$I = \oint_S \bar{J} \cdot d\bar{S} \rightarrow ①$$

→ Current I is constituted (made stable) due to outward flow of positive charges from the closed surface S.

→ According to principle of conservation of charge, there must be decrease of an equal amount of positive charge inside the closed surface.

→ ∴ outward rate of flow of positive charge gets balanced by the rate of decrease of charge inside the closed surface.

Let Q_i → charge within the closed surface,

$-\frac{dQ_i}{dt}$ → Rate of decrease of charge inside the closed surface.

→ Due to principle of conservation of charge, this $-\frac{dQ_i}{dt}$ is same as the rate of outward flow of charge, which is current.

$$\boxed{I = \oint \bar{J} \cdot d\bar{S} = -\frac{dQ_i}{dt}} \rightarrow ②$$

This is the integral form of continuity equation of current.

If the current is entering the volume,

$$\oint \bar{J} \cdot d\bar{s} = -I = +\frac{dQ_i}{dt} \rightarrow ③$$

using Divergence theorem,

$$\oint \bar{J} \cdot d\bar{s} = \int_{vol} (\nabla \cdot \bar{J})\, dv \rightarrow ④$$

$$-\frac{dQ_i}{dt} = \int_{vol} (\nabla \cdot \bar{J})\, dv \rightarrow ⑤$$

W.K.T. $Q_i = \int_{vol} \rho_v\, dv \rightarrow ⑥$

Sub ⑥ in ⑤

$$-\frac{d}{dt}\left[\int_{vol} \rho_v\, dv\right] = \int_{vol} (\nabla \cdot \bar{J})\, dv \rightarrow ⑦$$

$$\int_{vol} \nabla \cdot \bar{J}\, dv = -\frac{d}{dt}\left[\int_{vol} \rho_v\, dv\right]$$

$$\int_{vol} (\nabla \cdot \bar{J})\, dv = \int_{vol} -\frac{\partial \rho_v}{\partial t}\, dv$$

– For Incremental volume, Δv

$$(\nabla \cdot \bar{J})\, \Delta v = -\frac{\partial \rho_v}{\partial t}\, \Delta v$$

$$\boxed{\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}} \rightarrow ⑧$$ This is the

Point or Differential form of the Continuity equation of current.

For Steady (constant (uniform)) currents, which are not the functions of time, $\frac{\partial \rho_v}{\partial t} = 0$.

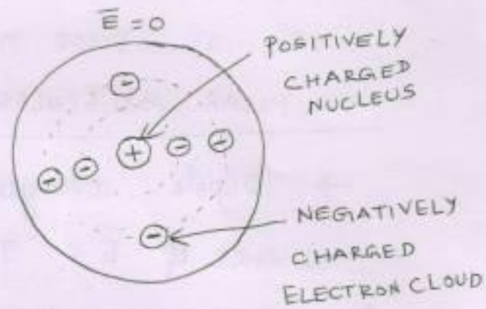
$$\boxed{\nabla \cdot \bar{J} = 0} \rightarrow ⑨$$ $\Rightarrow$ Rate of flow of charge remains constant with time.

POLARIZATION :

(i) UNPOLARIZED ATOM → When $\bar{E}$ is not applied to an atom of dielectric, (non conducting) number of positive charges is same as the negative charges & hence the atom is electrically neutral.

→ Positively charged nucleus is present at the center and negatively charged electrons are revolving around the nucleus in orbits (circles).

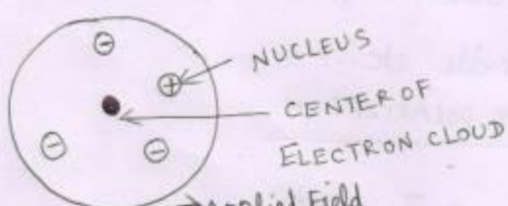
UNPOLARIZED ATOM OF A DIELECTRIC



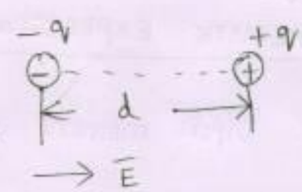
→ All the Negatively charged electrons are in the form of electron cloud.

(ii) POLARIZED ATOM : → To reduce the Electromotive force

→ When $\bar{E}$ is applied, Symmetrical distribution of charges gets disturbed.



Equivalent dipole



→ Positive charges experience a force $\bar{F} = Q\bar{E}$

→ Negative charges experience a force
$$\bar{F} = -Q\bar{E}$$

→ Now the atom is called polarized atom

in which there is the separation between the nucleus and the centre of electron cloud.

→ Polarization of dielectrics is the process in which the dipole gets aligned with the applied field.

TYPES OF DIELECTRICS:

1) NON POLAR MOLECULES

2) POLAR MOLECULES

NON POLAR MOLECULES

→ Dipole arrangement is totally absent in the absence of $\bar{E}$. It results only when E is applied.

eg: Hydrogen, oxygen and rare gases.

POLAR MOLECULES

→ Dipole arrangement exists without application of $\bar{E}$, but with random orientation. Under application of $\bar{E}$, dipole align with the direction of $\bar{E}$ eg water, sulphur dioxide etc...

MATHEMATIC EXPRESSION FOR POLARIZATION

Dipole moment, $\bar{P} = Q\bar{d} \rightarrow ①$

Where $Q \rightarrow$ Magnitude of one of 2 charges,

$\bar{d} \rightarrow$ Distance vector from negative to positive charge.

Total dipole moment, $\overline{P}_{total} = Q_1 \overline{d_1} + Q_2 \overline{d_2} + \cdots + Q_n \overline{d_n}$

$$\text{Total Dipole moment} = \sum_{i=1}^{n\Delta V} Q_i \overline{d_i} \rightarrow (2)$$

where $n \rightarrow$ no of dipoles / unit volume.

$\Delta V \rightarrow$ Total volume of the dielectric.

POLARIZATION : Polarization ($\overline{P}$) is defined as the total dipole moment / unit volume.

$$\overline{P} = \lim_{\Delta V \to 0} \frac{\sum_{i=1}^{n\Delta V} Q_i \overline{d_i}}{\Delta V} \rightarrow (3)$$

Flux Density in Dielectric is

$$\overline{D} = \varepsilon_0 \overline{E} + \overline{P} \rightarrow (4)$$

For isotropic & Linear medium, $\overline{P}$ & $\overline{E}$ are parallel to each other at every point & related to each other.

$$\overline{P} = X_e \varepsilon_0 \overline{E} \rightarrow (5)$$

where $X_e \rightarrow$ Electric susceptibility.

Sub (5) in (4).

$$\overline{P} = X_e \varepsilon_0 \overline{E} \text{ in } (4)$$

$$\overline{D} = \varepsilon_0 \overline{E} + X_e \varepsilon_0 \overline{E} = \overline{E} \varepsilon_0 (1 + X_e)$$

where $1 + X_e = \varepsilon_R \Rightarrow$ Relative permittivity or Dielectric constant of dielectric material.

$$\overline{D} = \overline{E} \varepsilon_0 \varepsilon_R = \overline{E} \varepsilon$$

$\boxed{\overline{D} = \overline{E} \varepsilon}$ $\quad \varepsilon \rightarrow$ Permittivity of dielectric.

PROPERTIES OF DIELECTRIC MATERIALS:

1) Due to Polarization, dielectrics can store the energy.

2) Due to Polarization, flux density of dielectric increases by amount equal to polarization

3) The electric field outside and inside the

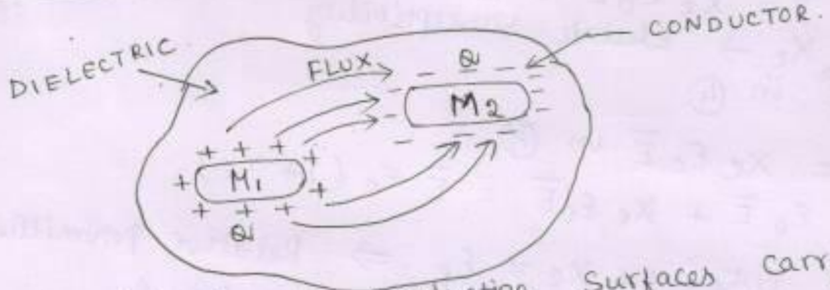
dielectric gets modified due to the induced electric dipoles.

DIELECTRIC STRENGTH:

→ Under large electric field, dielectric becomes conducting due to the presence of large number of free electrons. This condition is called dielectric breakdown.

→ Minimum value of the applied electric field at which the dielectric breaks down is called dielectric strength of the dielectric.

CONCEPT OF CAPACITANCE:



A System which has 2 Conducting Surfaces carrying equal and opposite charges, separated by a dielectric is called capacitive system giving rise to a capacitance.

Capacitance of two conductor system.

→ Ratio of Magnitudes of total charge on any one of the two conductors and potential Difference between the conductors is called Capacitance of two conductor System.

$$\boxed{C = \frac{Q}{V_{12}}}$$

where $Q \rightarrow$ charge in coulombs

$V \rightarrow$ potential difference in volts.

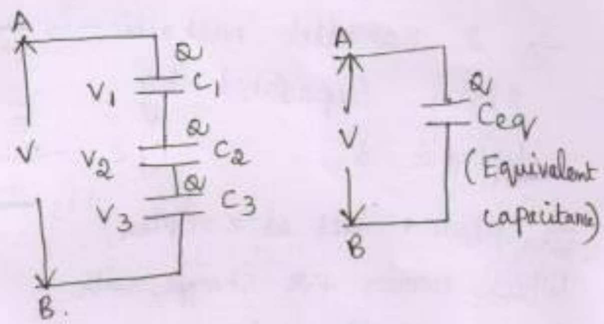
$$C = \frac{Q}{V} = \frac{\oint_S \varepsilon \bar{E} \cdot d\bar{s}}{-\int_{-}^{+} \bar{E} \cdot d\bar{L}} \text{ Farad}$$

where $Q = \oint_S \varepsilon \bar{E} \cdot d\bar{s} \rightarrow$ Gauss law

$V \rightarrow$ workdone in moving unit +ve charge from -ve to +ve surface

CAPACITORS IN SERIES:

$$Q = C_1V_1 = C_2V_2 = C_3V_3 \rightarrow (1)$$



$$V_1 = \frac{Q}{C_1} \; ; \; V_2 = \frac{Q}{C_2} \; ; \; V_3 = \frac{Q}{C_3}$$

$$C_{eq} = \frac{Q}{V} \; ; \; V = \frac{Q}{C_{eq}}$$

$$\boxed{V = V_1 + V_2 + V_3}$$

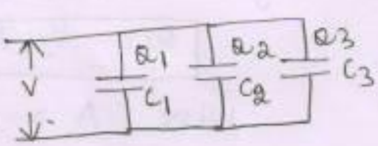
$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

For n capacitors in series $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$

CAPACITORS IN PARALLEL

$$Q = Q_1 + Q_2 + Q_3 \rightarrow (1)$$



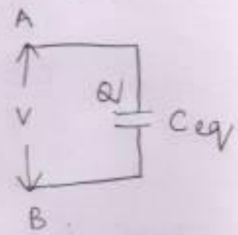
$$Q_1 = C_1V \; ; \; Q_2 = C_2V \; ; \; Q_3 = C_3V$$

$$Q = Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V = V(C_1 + C_2 + C_3)$$

$$Q = (C_1 + C_2 + C_3)V$$

$Q = C_{eq} \cdot V$

where $C_{eq} = C_1 + C_2 + C_3$

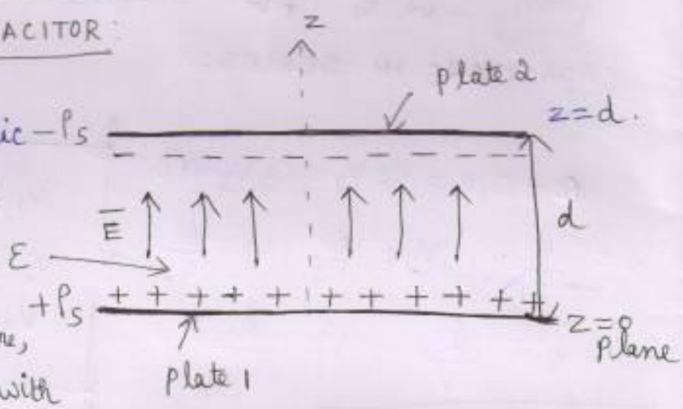


$Q = (C_1 + C_2 + C_3) V$

For 'n' capacitors in parallel

$C_{eq} = C_1 + C_2 + \cdots + C_n$

PARALLEL PLATE CAPACITOR



→ 2 parallel metallic plates separated by distance 'd'.

→ plate 1 lies at z=0 plane, which carries +ve charge with charge density $+\rho_s$

→ Plate 2 lies at z=d plane, which carries -ve charge with charge density $-\rho_s$

→ Dielectric presents between two plates with Permittivity (ε)

→ Magnitude of charge on any one plate,

$$\boxed{Q = \rho_s A \text{ coulomb (C)}}$$

where A → area of cross section of the plates in m^2.

ρ_s → Surface charge density.

→ Magnitude of charge in both the plates are equal.

TO FIND POTENTIAL DIFFERENCE

$$V = -\int_{-}^{+} \bar{E} \cdot d\bar{L}$$

where $\bar{E} = \bar{E}_1 + \bar{E}_2$

$$\bar{E}_1 = \frac{\rho_s}{2\varepsilon} \bar{a}_N = \frac{\rho_s}{2\varepsilon} \bar{a}_z \ V/m \Rightarrow$$ For infinite sheet of charge - plate 1.

$$\bar{E}_2 = -\frac{\rho_s}{2\varepsilon} \bar{a}_N = \frac{-\rho_s}{2\varepsilon} (-\bar{a}_z) \ V/m \Rightarrow$$ for infinite sheet of charge - plate 2.

→ $\bar{E}_1$ is normal at the boundary between conductor and dielectric without any tangential component

→ Direction of $\bar{E}_2$ is downwards in $-\bar{a}_z$ direction.

$$\bar{E} = \bar{E}_1 + \bar{E}_2 = \frac{\rho_s}{2\varepsilon} \bar{a}_z + \frac{\rho_s \bar{a}_z}{2\varepsilon}.$$

$$\bar{E} = \cancel{2} \frac{\rho_s}{\cancel{2}\varepsilon} \bar{a}_z \ ; \ \boxed{\bar{E} = \frac{\rho_s}{\varepsilon} \bar{a}_z}$$

Potential Difference is

$$V = -\int_{-}^{+} \bar{E} \cdot d\bar{L} = -\int_{\text{upper}}^{\text{Lower}} \frac{\rho_s}{\varepsilon} \bar{a}_z \cdot d\bar{L}$$

In cartesian system $d\bar{L} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z$

$$V = -\int_{z=d}^{z=0} \frac{\rho_s}{\varepsilon} \bar{a}_z \cdot [dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z]$$

$$= -\int_{z=d}^{z=0} \frac{\rho_s}{\varepsilon} dz .$$

$$= -\frac{\rho_s}{\varepsilon}\left[z\right]_d^0 = -\frac{\rho_s}{\varepsilon}\left[0-d\right]$$

$$\boxed{V = \frac{\rho_s d}{\varepsilon} \text{ volts (V)}}$$

Capacitance (C) The capacitance is the ratio of charge Q to the voltage V.

$$C = \frac{Q}{V} = \frac{\rho_s A}{\frac{\rho_s d}{\varepsilon}} = \frac{A\varepsilon}{d} \text{ Farad.}$$

$$C = \frac{A\varepsilon}{d} \text{ F}$$

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

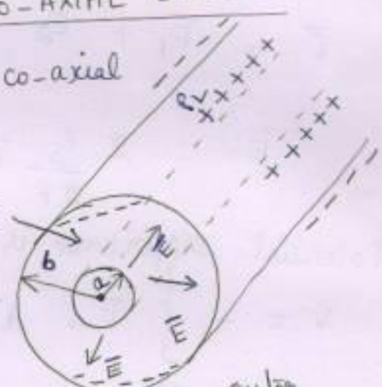
$$\boxed{C = \frac{A \varepsilon_0 \varepsilon_r}{d} \text{ F}}$$

CO-AXIAL CAPACITORS (OR) CO-AXIAL CABLE

Consider coaxial cable or co-axial capacitor.

'a' → Inner radius

'b' → outer radius



Inner conductor carries a charge density $+\rho_L$ C/m on its Surface & outer conductor carries a charge density $-\rho_L$ C/m.

→ Two conductors are separated by dielectric of permittivity ε

$$Q = \rho_L L$$

where L → length of the cable is L m.

Assume, cylindrical co-ordinate system,

For infinite line charge $\bar{E} = \frac{\rho_L}{2\pi\varepsilon r} \bar{a}_r \rightarrow (2)$

Find Potential Difference $V = -\int_{-}^{+} \bar{E} \cdot d\bar{l}$ consider ($d\bar{l}$ in radial direction $dr\,\bar{a}_r$)

$$V = -\int_{r=b}^{r=a} \frac{\rho_L}{2\pi\varepsilon r} \bar{a}_r \cdot dr\,\bar{a}_r$$

$$= -\frac{\rho_L}{2\pi\varepsilon} \int_{r=b}^{r=a} \frac{1}{r} \cdot dr\ (1)$$

$$V = -\frac{\rho_L}{2\pi\varepsilon} \left[\ln r\right]_{r=b}^{r=a} \quad ; \quad V = -\frac{\rho_L}{2\pi\varepsilon}\left[\ln a - \ln b\right]$$

$$V = \frac{\rho_L}{2\pi\varepsilon}\left[-\ln a + \ln b\right] ; \quad V = \frac{\rho_L}{2\pi\varepsilon}\left[\ln b - \ln a\right]$$

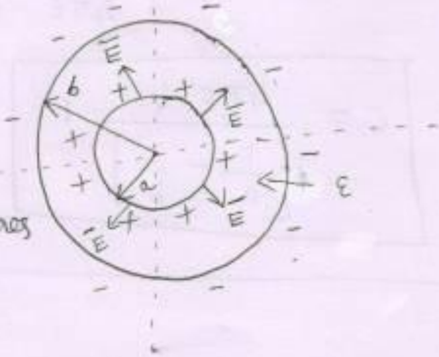
$$\boxed{V = \frac{\rho_L}{2\pi\varepsilon} \ln\left[\frac{b}{a}\right] \text{ V}}$$

$$C = \frac{Q}{V} = \frac{\cancel{\rho_L} \times L}{\frac{\cancel{\rho_L}}{2\pi\varepsilon} \ln\left[\frac{b}{a}\right]} \quad ; \quad \boxed{C = \frac{2\pi\varepsilon L}{\ln\left[\frac{b}{a}\right]} \text{ F}}$$

This is the capacitance of cylindrical capacitor.

SPHERICAL CAPACITORS

Consider spherical capacitor.



→ Inner sphere radius 'a'.

→ outer sphere radius 'b'

where $b > a$

→ Region between two spheres is filled with dielectric of permittivity ε.

→ Inner sphere is given a + charge,

→ outer sphere is given a − charge.

$\bar{E}$ in radial direction of Gaussian Surface as a sphere of radius r.

$$\bar{E} = \frac{Q}{4\pi \varepsilon r^2} \bar{a}_r \left(\frac{V}{m}\right).$$

$$V = -\int_{-}^{+} \bar{E} \cdot d\bar{L} = -\int_{r=b}^{r=a} \frac{Q}{4\pi\varepsilon r^2} \bar{a}_r \cdot d\bar{L} \rightarrow (2)$$

where $d\bar{L} = dr\, \bar{a}_r \rightarrow (3)$

$$V = -\int_{r=b}^{r=a} \frac{Q}{4\pi\varepsilon r^2} \bar{a}_r \cdot dr\, \bar{a}_r \qquad (\bar{a}_r \cdot \bar{a}_r = 1)$$

$$= -\frac{Q}{4\pi\varepsilon} \int_{r=b}^{r=a} r^{-2}\, dr = \frac{-Q}{4\pi\varepsilon} \left[\frac{r^{-1}}{-1}\right]_{r=b}^{r=a}$$

$$V = +\frac{Q}{4\pi\varepsilon} \left[\frac{1}{r}\right]_b^a = \frac{Q}{4\pi\varepsilon} \left[\frac{1}{a} - \frac{1}{b}\right]$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon} \left[\frac{1}{a} - \frac{1}{b}\right]}$$

$$C = \frac{Q}{V} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi\varepsilon}\left[\frac{1}{a} - \frac{1}{b}\right]}$$

$$\boxed{C = \frac{4\pi\varepsilon}{\left[\frac{1}{a} - \frac{1}{b}\right]} F}$$

CAPACITANCE OF SINGLE ISOLATED SPHERE:

→ consider a single isolated sphere of radius 'a' given a charge $+Q$.

→ It forms a capacitance with an outer plate which is infinitely large hence $b = \infty$

$$C = \frac{4\pi\varepsilon}{\left[\frac{1}{a} - \frac{1}{b}\right]} \text{ F}$$

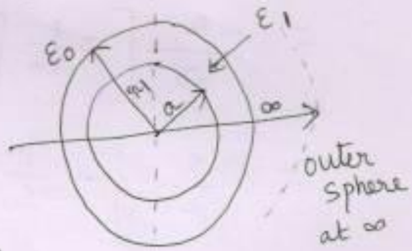
$$\frac{1}{\infty} = 0$$

$$C = \frac{4\pi\varepsilon}{\left[\frac{1}{a} - \frac{1}{\infty}\right]}$$

$$\boxed{C = 4\pi\varepsilon a \text{ F}}$$

ISOLATED SPHERE COATED WITH DIELECTRIC

→ consider a single isolated sphere coated with a dielectric having permittivity ε_1 upto radius r_1



→ Radius of Inner sphere is 'a'

Since it is placed in free space, outside the sphere $\varepsilon = \varepsilon_0$

It carries a +ve charge $(+Q)$.

So $\varepsilon = \varepsilon_1$ for $a < r < r_1$

$\varepsilon = \varepsilon_0$ for $r > r_1$

$$V = -\int_{-}^{+} \bar{E} \cdot d\bar{L} = -\int_{r=\infty}^{r=a} \bar{E} \cdot d\bar{L}$$

$$V = -\int_{r=\infty}^{r=a} \bar{E} \cdot d\bar{L} = -\int_{r=\infty}^{r=r_1} \bar{E} \cdot d\bar{L} - \int_{r=r_1}^{r=a} \bar{E} \cdot d\bar{L} \rightarrow (1)$$

For $a < r < r_1$, $\bar{E}_1 = \dfrac{Q}{4\pi \varepsilon_1 r^2} \bar{a}_r \rightarrow$ ②

For $r_1 < r < \infty$, $\bar{E}_2 = \dfrac{Q}{4\pi \varepsilon_0 r^2} \bar{a}_r \rightarrow$ ③

$d\bar{L} = dr\, \bar{a}_r$ Since E_1 & $\bar{E}_2$ are in radial direction.

$$V = -\int_{r=\infty}^{r=r_1} \frac{Q}{4\pi \varepsilon_0 r^2} \bar{a}_r \cdot dr\, \bar{a}_r - \int_{r=r_1}^{r=a} \frac{Q}{4\pi \varepsilon_1 r^2} \bar{a}_r \cdot dr \bar{a}_r$$

$$= -\frac{Q}{4\pi} \left[\frac{1}{\varepsilon_0} \int_{r=\infty}^{r_1} \frac{1}{r^2}\, dr + \frac{1}{\varepsilon_1} \int_{r=r_1}^{a} \frac{1}{r^2}\, dr \right]$$

$$= -\frac{Q}{4\pi} \left[-\frac{1}{\varepsilon_0} \left[\frac{1}{r} \right]_{\infty}^{r_1} - \frac{1}{\varepsilon_1} \left[\frac{1}{r} \right]_{r_1}^{a} \right]$$

$$= \frac{-Q}{4\pi} \left[-\frac{1}{\varepsilon_0} \left[\frac{1}{r_1} - \frac{1}{\infty} \right] - \frac{1}{\varepsilon_1} \left[\frac{1}{a} - \frac{1}{r_1} \right] \right]$$

$$= +\frac{Q}{4\pi} \left[+\frac{1}{\varepsilon_0} \left[\frac{1}{r_1} \right] + \frac{1}{\varepsilon_1} \left[\frac{1}{a} - \frac{1}{r_1} \right] \right]$$

$$V = \frac{Q}{4\pi} \left[\frac{1}{\varepsilon_0 r_1} + \frac{1}{\varepsilon_1} \left[\frac{1}{a} - \frac{1}{r_1} \right] \right]$$

$$C = \frac{Q}{V} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi} \left[\frac{1}{\varepsilon_0 r_1} + \frac{1}{\varepsilon_1} \left(\frac{1}{a} - \frac{1}{r_1} \right) \right]}$$

$$\boxed{C = \frac{4\pi}{\left[\frac{1}{\varepsilon_1} \left(\frac{1}{a} - \frac{1}{r_1} \right) + \frac{1}{\varepsilon_0 r_1} \right]}}$$

$$\frac{1}{C} = \frac{\left[\frac{1}{\varepsilon_1} \left(\frac{1}{a} - \frac{1}{r_1} \right) + \frac{1}{\varepsilon_0 r_1} \right]}{4\pi}$$

$$\frac{1}{C} = \left(\frac{\left[\frac{1}{a} - \frac{1}{r_1}\right]}{4\pi \varepsilon_1} + \frac{1}{4\pi\varepsilon_0 r_1} \right) \rightarrow (1)$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \rightarrow (2)$$

compare (1) & (2) we get

$$C_1 = \frac{4\pi \varepsilon_1}{\left[\frac{1}{a} - \frac{1}{r_1}\right]} \Rightarrow \text{capacitance of spherical capacitor}$$

$$C_2 = 4\pi\varepsilon_0 r_1 \Rightarrow \text{capacitance of isolated sphere}$$

CONDUCTORS

- → Drifting of electrons
- → Drift velocity
- → Drift current
- → Mobility

→ DRIFTING OF ELECTRONS: In Drifting of Electrons is meant for: Under the effect of the applied electric field, the available free electrons start moving. The moving electrons strike the adjacent atoms and rebound in the random directions. This is called drifting of electrons.

→ DRIFT VELOCITY: After some time, the electrons attain the constant average velocity called drift velocity (V_d).

DRIFT CURRENT: The current constituted due to the drifting of electrons in metallic conductors is called drift current.

MOBILITY: The Drift velocity is directly proportional to the applied electric field.

$$\overline{V_d} \propto \overline{E}$$

$$\overline{V_d} = -\mu_e \overline{E}$$

μ_e denote constant of proportionality is called mobility of the electrons.

→ −ve Sign indicates that the velocity of electrons is against the direction of electric field $\overline{E}$

$$\mu = \frac{\text{velocity}}{\text{field}} = \frac{m/s}{V/m} = \frac{m^2}{V\text{-}s}$$

Mobility is measured in square metres per volt second.

Value of Mobility:

For aluminium 0.0012.

For copper 0.0032.

Current Density $\overline{J} = \rho_e \overline{V}$

number of Protons & Electrons is same & it is always electrically neutral

$\rho_v = 0$. For neutral Material.

Drift velocity : Drift is defined as the velocity of free electrons.

$$\overline{J} = \rho_e \overline{v_d}$$

where $\rho_e \rightarrow$ charge density due to free electrons.

$v_d \rightarrow$ Drift velocity

$J \rightarrow$ current density.

$$\overline{J} = \rho_e(-\mu_e \overline{E})$$

$$\boxed{\overline{J} = -\rho_e \mu_e \overline{E}}$$

POINT FORM OF OHM'S LAW

point form of Ohm's Law equation

$\overline{J} = \sigma \overline{E}$ For a metallic conductor.

where $E \rightarrow$ Electric field.
$\sigma \rightarrow$ Conductivity of the material $\left(\frac{\mho}{m}\right)$
$J \rightarrow$ current density

$\sigma = -\rho_e \mu_e$

where $\rho_e \rightarrow$ charge density due to free electrons.
$\mu_e \rightarrow$ Mobility of free electrons.

If temperature increases, Vibrations of crystalline structure of atoms increases $\Rightarrow$ Drift velocity decreases $\Rightarrow$ mobility decreases $\Rightarrow$ conductivity decreases $\Rightarrow$ Resistivity increases.

POISSON'S AND LAPLACE'S EQUATIONS

$\rightarrow$ These equations are used for solving boundary value problems.

$\rightarrow$ Boundary value problems $\rightarrow$ practically charge & potential values may be known at some boundaries of the region. From those values, it is necessary to obtain potential and $\bar{E}$ throughout the region.

Applications

POISSON'S & LAPLACE'S EQUATION:

To solve boundary value problems.

DERIVATION FOR POISSON'S & LAPLACE'S EQUATION:

point form of Gauss law,

$$\nabla \cdot \bar{D} = \rho_V \rightarrow (1)$$

where

$\bar{D} \rightarrow$ Flux Density

$\rho_V \rightarrow$ volume charge density

$$\bar{D} = \varepsilon \bar{E}$$

Sub $\bar{D}$ in (1)

$\nabla \cdot \varepsilon \bar{E} = \rho_v$

$\bar{E} = -\nabla V$ (from Gradient relationship)

$\boxed{\nabla \cdot \varepsilon(-\nabla V) = \rho_v}$ → For inhomogeneous medium for which ε is not constant

$-\varepsilon[\nabla \cdot \nabla V] = \rho_v$; $\nabla \cdot \nabla V = -\frac{\rho_v}{\varepsilon}$

$\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon}}$ → For homogeneous medium, for which ε is constant

Laplace's Equation Special case of poisson's equation.

→ volume charge density $\rho_v = 0$ (ie) charge free region

$$\boxed{\nabla^2 V = 0.}$$

$\nabla^2 V$ → Laplacian of V.

LAPLACIAN OPERATION IN DIFFERENT CO-ORDINATE SYSTEMS

① Cartesian co-ordinate system

Laplace's equation in cartesian form

$$\nabla V = \frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z$$

$$\nabla^2 V = \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial V}{\partial z}\right)$$

$$\boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0}$$

(2) CYLINDRICAL CO-ORDINATE SYSTEM:

Laplace's equation in cylindrical form.

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2} = 0.$$

(3) SPHERICAL CO-ORDINATE SYSTEM:

Laplace's equation in Spherical form

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0.$$

PROCEDURE FOR SOLVING LAPLACE'S EQUATION

1. use method of integration solve Laplace's equation. Assume constants of integration.
2. Determine constants by applying the boundary conditions given solution obtained from step 1 & step 2 is unique solution.
3. obtain $\bar{E} = -\nabla V$
4. obtain $\bar{D} = \varepsilon \bar{E}$ for homogenous medium.
5. obtain charge induced on the conductor surface, $Q = -\int \rho_s \, ds$, where $\rho_s = D_N$ (Normal component to the surface)
6. Calculate Capacitance(C) of the system

1) Determine whether or not the following potential fields satisfy the Laplace's equation. $\Rightarrow$ $\boxed{\nabla^2 V = 0}$

a) $V = x^2 - y^2 + z^2$

$\nabla^2 V = \;?$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$= \frac{\partial^2}{\partial x^2}[x^2 - y^2 + z^2] + \frac{\partial^2}{\partial y^2}(x^2 - y^2 + z^2) + \frac{\partial^2}{\partial z^2}(x^2 - y^2 + z^2)$$

$$= \cancel{2} - \cancel{2} + 2$$

$\nabla^2 V = 2$

Since $\nabla^2 V \neq 0$, Field V does not satisfy Laplace's equation.

b) $V = r \cos\phi + z$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\frac{\partial V}{\partial r} = \cos\phi \; ; \; \frac{\partial V}{\partial \phi} = -r \sin\phi \; ; \; \frac{\partial V}{\partial z} = 1$$

$$\frac{\partial^2 V}{\partial \phi^2} = -r \cos\phi \; ; \; \frac{\partial^2 V}{\partial z^2} = 0$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}(r \cos\phi) + \frac{1}{r^2}(-r \cos\phi) + 0$$

$$= \frac{1}{r}\cos\phi - \frac{\cos\phi}{r}$$

$$= 0$$

$\nabla^2 V = 0$

Given field V satisfies Laplace's equation.

c) $V = r\cos\theta + \phi$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2 V}{\partial\phi^2}\right)$$

$$\frac{\partial V}{\partial r} = \cos\theta \; ; \quad \frac{\partial V}{\partial\theta} = -r\sin\theta \; ; \quad \frac{\partial V}{\partial\phi} = 1$$

$$\frac{\partial^2 V}{\partial\phi^2} = 0$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cos\theta\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\cdot(-r\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}(0).$$

$$\nabla^2 V = \frac{1}{r^{\cancel{2}}}\left[2\cancel{r}\cos\theta\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[-r\sin^2\theta\right] + 0.$$

$$= \frac{2}{r}\cos\theta - \frac{\cancel{r}}{2r^{\cancel{2}}\sin\theta}\frac{\partial}{\partial\theta}\left[1-\cos 2\theta\right]$$

$$= \frac{2}{r}\cos\theta - \frac{1}{\cancel{2}r\sin\theta}\left[0 + \sin 2\theta\cdot\cancel{2}\right]$$

$$= \frac{2}{r}\cos\theta - \frac{\sin 2\theta}{r\sin\theta}$$

$$= \frac{2}{r}\cos\theta - \frac{2\cancel{\sin\theta}\cos\theta}{r\cancel{\sin\theta}}$$

$$= \frac{2}{r}\cos\theta - \frac{2}{r}\cos\theta = 0$$

$$\boxed{\begin{array}{c} \cos 2\theta = 1-2\sin^2\theta \\ 1-\cos 2\theta = 2\sin^2\theta \\ \frac{1-\cos 2\theta}{2} = \sin^2\theta \\ \sin 2A = 2\sin A\cos A \\ \sin 2\theta = 2\sin\theta\cos\theta \end{array}}$$

$\nabla^2 V = 0$

Given field V Satisfies Laplace's equation.

1) Verify that the potential field given below satisfies Laplace's equation, $V = 2x^2 - 3y^2 + z^2$.

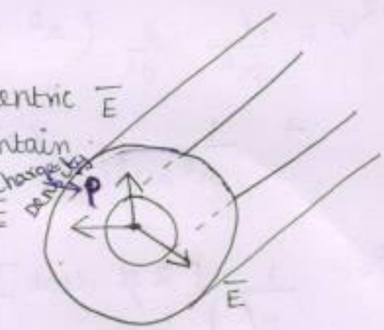
$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$= 4 - 6 + 2 = 0$$

Given field satisfies the Laplace's equation.

POISSON'S EQUATION

The region between two concentric right circular cylinders contain a uniform charge density ρ. Solve the poisson's equation for the potential in the region.



Consider cylindrical co-ordinate system. In the given structure, $\bar{E}$ is in radial direction from inner to outer cylinder.

Here, both $\bar{E}$ & V are functions of r only & not of ϕ & z

CYLINDRICAL CO-ORDINATE SYSTEM

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r \cdot \frac{\partial V}{\partial r}\right) \quad \left[\because \frac{\partial V}{\partial \phi} \ \& \ \frac{\partial V}{\partial z} \text{ are equal to zero}\right]$$

According to poisson's equation,

$$\nabla^2 V = -\frac{\rho}{\varepsilon}$$

$$-\frac{\rho}{\varepsilon} = \frac{1}{r}\frac{\partial}{\partial r}\left(r \cdot \frac{\partial V}{\partial r}\right)$$

$$-\frac{\rho r}{\varepsilon} = \frac{\partial}{\partial r}\left(r \cdot \frac{\partial V}{\partial r}\right)$$

Integrate on both sides, we get

$$\int \frac{\partial}{\partial r}\left(r \cdot \frac{\partial V}{\partial r}\right) = -\int \frac{\rho r}{\varepsilon}\, dr$$

$$r \cdot \frac{\partial V}{\partial r} = -\frac{\rho}{\varepsilon}\left[\frac{r^2}{2}\right] + C_1$$

$$\frac{\partial V}{\partial r} = -\frac{\rho}{\varepsilon \cancel{r}}\left[\frac{r^{\cancel{2}}}{2}\right] + \frac{C_1}{r}$$

$$\frac{\partial V}{\partial r} = -\frac{\rho r}{2\varepsilon} + \frac{C_1}{r}$$

Integrate on both sides,

$$\int \frac{\partial V}{\partial r} = \int\left(-\frac{\rho r}{2\varepsilon} + \frac{C_1}{r}\right) dr$$

$$\boxed{V = -\frac{\rho}{2\varepsilon}\left[\frac{r^2}{2}\right] + C_1[\ln r] + C_2}$$

Problem:

1) In free space, $\rho_v = \frac{200\,\varepsilon_0}{r^{2.4}}$ (i) Use poisson equation, to find V as a function of r, if it is assumed that, $r^2 E_r \to 0$ as $r \to 0$ & $V \to 0$ as $r \to \infty$. Use spherical co-ordinate system (ii) Find potential V as a function of r using Gauss law & line Integral.

Solution

(i) According to poisson's equation

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon_0}$$

$\rho_v \rightarrow$ volume charge density

$\varepsilon_0 \rightarrow$ Permittivity of Free space

$$\nabla^2 V = -\frac{200\,\cancel{\varepsilon_0}}{r^{2.4}\,\cancel{\varepsilon_0}} = -\frac{200}{r^{2.4}}$$

$$\boxed{\nabla^2 V = -\frac{200}{r^{2.4}}}$$

Given that, V is a function of r only & not the function of θ & ϕ.

consider Spherical co-ordinate system,

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] = -\frac{200}{r^{2.4}}$$

$$\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] = -\frac{200\,r^2}{r^{2.4}}$$

$$\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] = -200\,r^{2-2.4} = -200\,r^{-0.4}$$

$$\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] = -200\,r^{-0.4}$$

Integrate both sides, we get

$$\int \frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = -\int 200\,r^{-0.4}\,dr$$

$$r^2\frac{\partial V}{\partial r} = -200\left(\frac{r^{-0.4+1}}{-0.4+1}\right) + c_1 = -200\frac{r^{0.6}}{(+0.6)} + c_1$$

$$\boxed{r^2\frac{\partial V}{\partial r} = -333.33\,r^{0.6} + c_1} \longrightarrow (1)$$

Since $\bar{E}$ is the Function of r only

$$\boxed{\bar{E} = -\nabla V}$$

$$= \left(-\frac{\partial V}{\partial r}\right)\bar{a}_r$$

$$\bar{E} = E_r\,\bar{a}_r$$

So $E_r = -\frac{\partial V}{\partial r}$; $\boxed{\frac{\partial V}{\partial r} = -E_r} \longrightarrow (2)$

Sub $\frac{\partial V}{\partial r} = -E_r$ in ①.

$$\boxed{r^2(-E_r) = -333.33\, r^{0.6} + C_1} \longrightarrow ③$$

Since $r \to 0$, $r^2 E_r \to 0$

$$0 = 0 + C_1$$

$$\boxed{C_1 = 0}$$

Sub $C_1 = 0$ in ① we get.

$$r^2 \frac{\partial V}{\partial r} = -333.33\, r^{0.6} + 0$$

$$\frac{\partial V}{\partial r} = -333.33\, r^{0.6-2} = -333.33\, r^{-1.4}$$

$$\frac{\partial V}{\partial r} = -333.33\, r^{-1.4}$$

Integrate on both sides, we get

$$\int \partial V = -\int 333.33\, r^{-1.4}\, dr$$

$$V = -333.33 \left[\frac{r^{-1.4+1}}{-0.4}\right] + C_2$$

$$V = \frac{333.33}{0.4}\, r^{-0.4} + C_2$$

$$\boxed{V = \frac{833.325}{r^{0.4}} + C_2}$$

use $V \to 0$ as $r \to \infty$

$$0 = \frac{833.325}{(\infty)^{0.4}} + C_2.$$

$$\boxed{C_2 = 0}$$

$$\boxed{V = \frac{833.325}{r^{0.4}}\ V.}$$

(ii) GAUSS'S LAW,

$$\nabla \cdot \bar{D} = \rho_v$$

Where $\bar{D} \rightarrow$ Electric Flux Density

$\rho_v \rightarrow$ Volume charge density

$\varepsilon_0 \rightarrow$ Permittivity of free space

$\bar{E} \rightarrow$ Electric field intensity

$$\nabla \cdot \varepsilon_0 \bar{E} = \rho_v$$

$$\nabla \cdot \bar{E} = \frac{\rho_v}{\varepsilon_0} = \left(\frac{200\,\cancel{\varepsilon_0}}{r^{2.4}}\right) \cdot \frac{1}{\cancel{\varepsilon_0}}$$

$$\boxed{\nabla \cdot \bar{E} = \frac{200}{r^{2.4}}} \rightarrow ①$$

Where $\bar{E} = E_r \bar{a}_r$ consider spherical co-ordinate system,

$$\nabla \cdot \bar{E} = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 E_r) \rightarrow ②$$

compare ① & ②

$$\frac{1}{r^2} \frac{\partial}{\partial r}(r^2 E_r) = \frac{200}{r^{2.4}}$$

$$\frac{\partial}{\partial r}(r^2 E_r) = \frac{200}{r^{2.4}} r^2 = \frac{200}{r^{2.4-2}} = \frac{200}{r^{0.4}} = 200\, r^{-0.4}$$

$$\frac{\partial}{\partial r}(r^2 E_r) = \frac{200}{r^{0.4}}.$$

Integrate on both sides

$$\int \frac{\partial}{\partial r}(r^2 E_r) = \int \frac{200}{r^{0.4}}$$

$$r^2 E_r = \int \frac{200}{r^{0.4}}\, dr = 200\left[\frac{r^{-0.4+1}}{0.6}\right] + c_1$$

$$r^2 E_r = 333.33\, r^{0.6} + c_1 \rightarrow ③$$

$r^2 E_r \rightarrow 0$ as $r \rightarrow 0$

$$0 = c_1 \; ; \quad \boxed{c_1 = 0}$$

sub $c_1 = 0$ in ③

$$r^2 E_r = 333.33\, r^{0.6}$$

$$E_r = 333.33\, r^{0.6-2} = 333.33\, r^{-1.4}.$$

$$E_r = 333.33\, r^{-1.4} \rightarrow ④$$

$$\bar{E} = E_r \bar{a}_r$$

sub E_r in ④

$$\boxed{\bar{E} = 333.33\, r^{-1.4}\, \bar{a}_r \;\; V/m}$$

$$V = -\int \bar{E} \cdot d\bar{L}$$

where $d\bar{L} = dr\, \bar{a}_r$

$\bar{E} = 333.33\, r^{-1.4}\, \bar{a}_r$

$$V = -\int (333.33\, r^{-1.4}\, \bar{a}_r) \cdot (dr\, \bar{a}_r)$$

$$= -\int 333.33\, r^{-1.4}\, dr = -333.33 \int r^{-1.4}\, dr$$

$$= -333.33 \left[\frac{r^{-1.4+1}}{-0.4}\right] + c_2$$

$$V = \frac{333.33}{0.4}\, r^{-0.4} + c_2$$

$$\boxed{V = \frac{833.325}{r^{0.4}} + c_2} \rightarrow (5)$$

$V = 0$ as $r \to \infty$; $0 = 0 + c_2$; $\boxed{c_2 = 0}$

Sub $c_2 = 0$ in (5), we get,

$$\boxed{V = \frac{833.325}{r^{0.4}}\ V}$$

BOUNDARY CONDITIONS FOR ELECTRIC FIELD:

BOUNDARY CONDITIONS:

→ conditions existing at the boundary of the two media when electric field passes from one medium to other

→ Depending on the nature of media, there are two types of boundary conditions.

TWO TYPES OF BOUNDARY CONDITIONS

1. Boundary between conductor and free space (or) Dielectric.
2. Boundary between two dielectrics with different properties.

PREREQUISITE TO STUDY BOUNDARY CONDITIONS

1. Maxwell's equations for electrostatics.

$\oint \bar{E} \cdot d\bar{L} = 0$; $\oint \bar{D} \cdot d\bar{S} = Q$.

E → Electric field intensity V/m

D → Electric Flux Density (C/m^2).

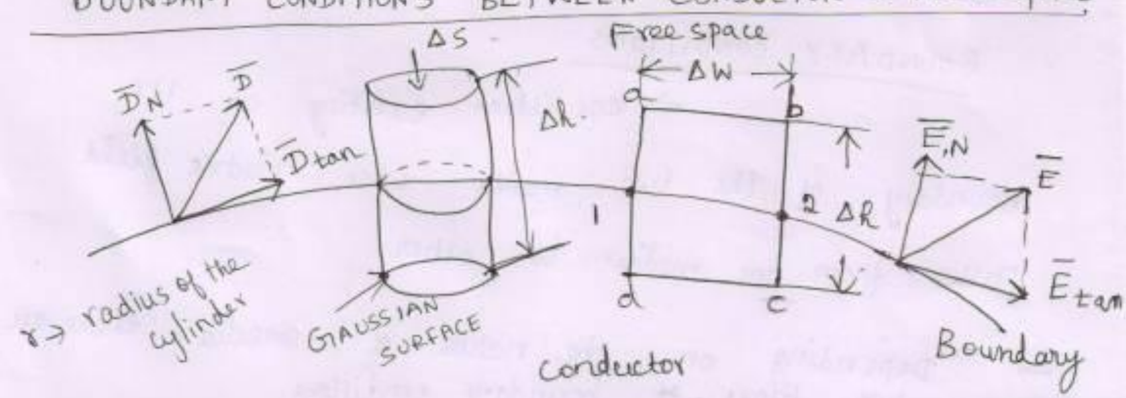
2. $\bar{E}$ in terms of two components.
   1. Tangential component &
   2. Normal component to the boundary.

$$\bar{E} = \bar{E}_{tan} + \bar{E}_N$$

$\bar{D}$ in terms of two components.
   1. Tangential Component &
   2. Normal component to the boundary

$$\bar{D} = \bar{D}_{tan} + \bar{D}_N$$

BOUNDARY CONDITIONS BETWEEN CONDUCTOR & FREE SPACE



To Find Boundary Conditions between conductor and free space. Let us consider closed Path & Gaussian Surface. II - (17)

1) $\bar{E}$ at the boundary

$$\bar{E} = \bar{E}_{tan} + \bar{E}_N$$

$\oint \bar{E} \cdot dL = 0 \Rightarrow$ work done in carrying unit positive charge along a closed path is zero.

Consider a Rectangular closed path, a-b-c-d-a.

$$\oint \bar{E} \cdot d\bar{L} = \int_a^b \bar{E} \cdot d\bar{L} + \int_b^c \bar{E} \cdot d\bar{L} + \int_c^d \bar{E} \cdot d\bar{L} + \int_d^a \bar{E} \cdot d\bar{L}$$

$$0 = \int_a^b \bar{E} \cdot d\bar{L} + \int_b^c \bar{E} \cdot d\bar{L} + 0 + \int_d^a \bar{E} \cdot d\bar{L}$$

[side c-d is in the conductor where $\bar{E} = 0$]

To find $\int_a^b \bar{E} \cdot d\bar{L}$

$$\int_a^b \bar{E} \cdot d\bar{L} = \bar{E} \int_a^b d\bar{L} = \bar{E}\, \Delta W = |\bar{E}_{tan}|\, \Delta W$$

$$\boxed{\int_a^b \bar{E} \cdot d\bar{L} = E_{tan}\, \Delta W}$$

$\Rightarrow$ Two sides a-b & c-d of closed contour are parallel to tangential direction to the surface.

$\Rightarrow$ Two sides b-c & d-a are normal direction to the surface.

$\Rightarrow$ Half of the rectangle is in conductor and remaining half of the rectangle is in free space.

Elementary height (Δh) $\rightarrow \frac{\Delta h}{2}$ is in conductor

$\rightarrow \frac{\Delta h}{2}$ is in free space.

Elementary weight (ΔW)

To find $\int_b^c \bar{E} \cdot d\bar{L}$

$$\int_b^c \bar{E} \cdot d\bar{L} = \bar{E} \int_b^c d\bar{L} = |\bar{E}_N| \int_b^c d\bar{L} = E_N \int_b^c d\bar{L}$$

$$\int_b^c d\bar{L} = \int_b^2 d\bar{L} + \int_2^c d\bar{L} = \frac{\Delta h}{2} + 0 = \frac{\Delta h}{2}$$

$$\boxed{\int_b^c \bar{E}\cdot d\bar{L} = E_N \frac{\Delta h}{2}}$$

to find $\int_d^a \bar{E}\cdot d\bar{L} = ?$

$$\int_d^a \bar{E}\cdot d\bar{L} = \int_d^1 \bar{E}\cdot d\bar{L} + \int_1^a \bar{E}\cdot d\bar{L}$$

$= 0 + \int_1^a \bar{E}\cdot d\bar{L}$ [portion (d-1) is in the conductor where, $\bar{E} \doteq 0$]

$$= \int_1^a |E_N|\, d\bar{L}$$

$$= E_N \int_1^a d\bar{L} = E_N\left(-\frac{\Delta h}{2}\right)$$

$$= -E_N \frac{\Delta h}{2}$$

$$\boxed{\int_d^a \bar{E}\cdot d\bar{L} = -E_N \frac{\Delta h}{2}}$$

$$E_{tan}\,\Delta W + \cancel{E_N \frac{\Delta h}{2}} - \cancel{E_N \frac{\Delta h}{2}} = 0$$

$$E_{tan}\,\Delta W = 0$$

$$\boxed{E_{tan} = 0.}$$

Tangential component of $\bar{E}$ is zero at the boundary between conductor and free space (ie) $\bar{E}$ is always in the direction $\perp$r to the boundary

$\bar{D} = \varepsilon_0 \bar{E}$ for free space

$D_{tan} = \varepsilon_0 E_{tan} = 0$

$\boxed{D_{tan} = 0}$

Tangential component of $\bar{D}$ is zero at the boundary between conductor & free space

$\Rightarrow$ $\bar{D}$ is always in the direction $\perp$r to the boundary.

2) DN at the boundary

To find the normal component of $\bar{D}$, Assume a closed Gaussian surface in the form of right circular cylinder for which $\frac{\Delta h}{2}$ is in conductor . & remaining $\frac{\Delta h}{2}$ is in Free space.

→ Its axis is normal to the direction of surface.

According to Gauss law, $\oint \bar{D} \cdot d\bar{s} = Q \rightarrow ①$

$$\oint \bar{D} \cdot d\bar{s} = \int_{top} \bar{D} \cdot d\bar{s} + \int_{bottom} \bar{D} \cdot d\bar{s} + \int_{lateral} \bar{D} \cdot d\bar{s} = Q \rightarrow ②$$

Bottom surface is in conductor, where $\bar{D} = 0$

$$\text{So, } \int_{bottom} \bar{D} \cdot d\bar{s} = 0 . \rightarrow ③$$

Lateral surface area is $2\pi r \Delta h$.

Where $r \rightarrow$ radius of the cylinder

$\Delta h \rightarrow$ elementary height.

if $\Delta h \rightarrow 0 \Rightarrow$ this area (Lateral surface area) reduces to zero.

$$\int_{lateral} \bar{D} \cdot d\bar{s} = 0 \quad \rightarrow (4)$$

sub (3) & (4) in (2)

$$0 + 0 + \int_{top} \bar{D} \cdot d\bar{s} = Q \; ; \quad \int_{top} \bar{D} \cdot d\bar{s} = Q$$

$$Q = \int_{top} D_N \, d\bar{s} = D_N \int_{top} d\bar{s}$$

From Gausslaw, $\boxed{Q = D_N \, \Delta s}$

$\boxed{Q = \rho_s \Delta s} \Rightarrow$ At the boundary, the charge exits in the form of surface charge density ρ_s C/m².

$$D_N \cancel{\Delta s} = \rho_s \cancel{\Delta s} \qquad \boxed{D_N = \rho_s} \rightarrow (5)$$

$$\boxed{\begin{aligned} D_N &= \rho_s \\ D_N &= \varepsilon_0 E_N \end{aligned}}$$

$$\rho_s = \varepsilon_0 E_N \; ; \quad \boxed{E_N = \frac{\rho_s}{\varepsilon_0}} \rightarrow (6)$$

<u>conductor</u>. has ∞ conductivity <u>eg:</u> copper, silver conductivity in the order of 10^6 S/m.

Electric Field intensity $\bar{E}$, $\bar{D}$ ← electric flux density, & charge density within the conductor, ρ_v are zero [No charge can exit within the conductor only the charge appears on the surface in the form of surface charge density ρ_s]

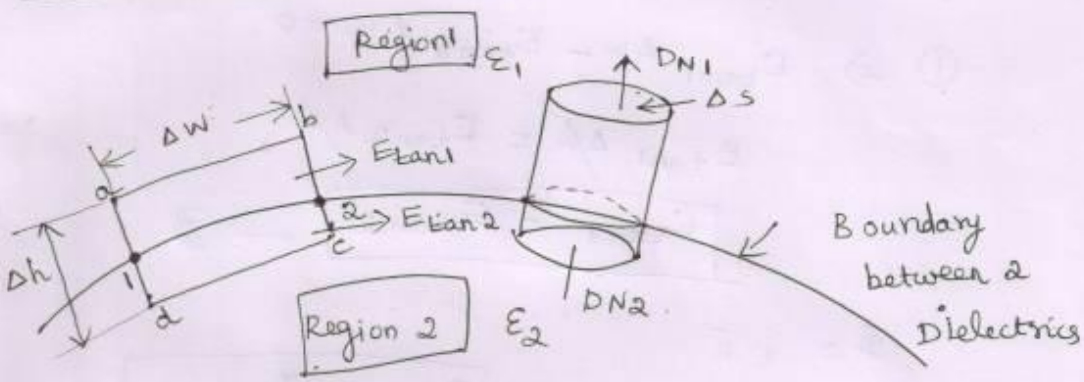
BOUNDARY CONDITIONS BETWEEN CONDUCTOR & DIELECTRIC

Free space is a dielectric with $\varepsilon = \varepsilon_0$.

If the boundary is between conductor & dielectric with $\varepsilon = \varepsilon_0 \varepsilon_r$

$$\boxed{\begin{array}{l} E_{tan} = D_{tan} = 0 \\ D_N = \rho_s \\ E_N = \dfrac{\rho_s}{\varepsilon} = \dfrac{\rho_s}{\varepsilon_0 \varepsilon_r} \end{array}}$$

BOUNDARY CONDITIONS BETWEEN 2 PERFECT DIELECTRICS



$\varepsilon_1 \rightarrow$ permittivity of dielectric 1

$\varepsilon_2 \rightarrow$ permittivity of dielectric 2

Elementary height $\rightarrow \Delta h$.

Elementary width $\rightarrow \Delta W$.

$\frac{\Delta h}{2}$ is in dielectric 1 & remaining

$\frac{\Delta h}{2}$ is in dielectric 2

$$\oint \bar{E} \cdot d\bar{L} = 0$$

$$\oint_a^b \bar{E} \cdot d\bar{L} + \oint_b^c \bar{E} \cdot d\bar{L} + \int_c^d \bar{E} \cdot d\bar{L} + \int_d^a \bar{E} \cdot d\bar{L} = 0.$$

case

$\Delta h \to 0$ integrals $\oint_b^c$ & $\int_d^a$ become zero.

$$\oint_a^b \bar{E} \cdot d\bar{L} + \int_c^d \bar{E} \cdot d\bar{L} = 0 \rightarrow (1)$$

$$\bar{E} \int_a^b d\bar{L} = E_{tan1} \int_a^b d\bar{L} = E_{tan1} \, \Delta w.$$

$$\int_c^d \bar{E} \cdot d\bar{L} = \bar{E} \int_c^d d\bar{L} = -E_{tan2} \, \Delta W$$

$$(1) \Rightarrow E_{tan1} \, \Delta W - E_{tan2} \, \Delta W = 0.$$

$$E_{tan1} \cancel{\Delta w} = E_{tan2} \cancel{\Delta w}$$

$$\boxed{E_{tan1} = E_{tan2}} \rightarrow (2)$$

$$\because \bar{D} = \varepsilon \bar{E}$$

$$D_{tan1} = \varepsilon_1 E_{tan1} \; ; \quad E_{tan1} = \frac{D_{tan1}}{\varepsilon_1}$$

$$D_{tan2} = \varepsilon_2 E_{tan2} \; ; \quad E_{tan2} = \frac{D_{tan2}}{\varepsilon_2} \rightarrow (3)$$

Sub (3) in (2)

$$\frac{D_{tan1}}{\varepsilon_1} = \frac{D_{tan2}}{\varepsilon_2}$$

$$\boxed{\frac{D_{tan1}}{D_{tan2}} = \frac{\varepsilon_1}{\varepsilon_2}} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

According to Gauss Law,

$$\oint \bar{D} \cdot d\bar{s} = Q$$

$$\left[\int_{top} + \int_{bottom} + \int_{lateral}\right] \bar{D} \cdot d\bar{s} = Q \rightarrow (1)$$

$$\int_{Lateral} \bar{D} \cdot d\bar{s} = 0 \quad \text{as} \quad \Delta h \to 0$$

$$\int_{top} \bar{D} \cdot d\bar{s} = D_{N1} \int_{top} d\bar{s} = D_{N1} \Delta S \rightarrow (2)$$

$$\int_{bottom} \bar{D} \cdot d\bar{s} = -D_{N2} \int_{bottom} d\bar{s} = -D_{N2} \Delta S$$

Sub (2) in (1)

$$D_{N1} \Delta S - D_{N2} \Delta S + 0 = Q \rightarrow (3)$$

$$Q = \rho_S \Delta S \rightarrow (4)$$

Equating (3) & (4) we can get

$$\rho_S \Delta S = D_{N1} \Delta S - D_{N2} \Delta S$$

$$\rho_S \cancel{\Delta S} = \cancel{\Delta S} (D_{N1} - D_{N2})$$

$$D_{N1} - D_{N2} = \rho_S$$

Put $\rho_S = 0$ At the ideal dielectric media boundary

$$D_{N1} - D_{N2} = 0$$

$$\boxed{D_{N1} = D_{N2}}$$

$$D_{N1} = \varepsilon_1 E_{N1} \qquad D_{N2} = \varepsilon_2 E_{N2}$$

$$\frac{D_{N1}}{D_{N2}} = \frac{\varepsilon_1 E_{N1}}{\varepsilon_2 E_{N2}} \qquad [D_{N1} = D_{N2}]$$

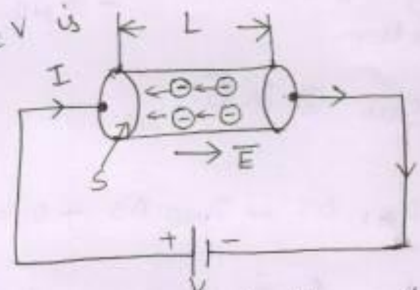
$$\frac{D_{N1}}{D_{N1}} = \frac{\varepsilon_1 E_{N1}}{\varepsilon_2 E_{N2}} = 1$$

$$\frac{E_{N1}}{E_{N2}} \frac{\varepsilon_1}{\varepsilon_2} = 1 ; \quad \boxed{\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}}}$$

Normal component of $\bar{E}$ is inversely proportional to relative permittivities of 2 media.

RESISTANCE OF A CONDUCTOR:

→ Consider that the voltage V is applied to a conductor of length L having S uniform cross-section.



→ The direction of $\bar{E}$ is same as the direction of conventional current, which is opposite to the flow of electrons.

Appy Electric Field

$$E = \frac{V}{L}.$$

conductor has uniform cross section S

$$I = \int_S \bar{J} \cdot d\bar{S} = JS$$

current direction is normal to the surface S.

$$J = \frac{I}{S} = \sigma E \qquad \text{where } \sigma \rightarrow \text{conductivity of the material}$$

$$\therefore J = \sigma\left(\frac{V}{L}\right) \qquad \left[E = \frac{V}{L}\right]$$

$$V = \frac{JL}{\sigma} = \left(\frac{I}{S}\right)\frac{L}{\sigma} = \left(\frac{L}{\sigma S}\right) I ; \quad \boxed{V = \left(\frac{L}{\sigma S}\right) I}$$

Sub Voltage (V)

$$R = \frac{V}{I} = \frac{L}{\sigma S}\frac{\cancel{I}}{\cancel{I}} \; ; \; \boxed{R = \frac{L}{\sigma S}}$$

For non-uniform fields

Resistance for non uniform fields is

$$\boxed{R = \frac{V_{ab}}{I} = \frac{-\int_b^a \bar{E}\cdot d\bar{L}}{\int_S \bar{J}\cdot d\bar{S}} = \frac{-\int_b^a \bar{E}\cdot d\bar{L}}{\int_S \sigma\bar{E}\cdot d\bar{S}}}$$

$$\boxed{R = \frac{L}{\sigma S} = \frac{\rho_c L}{S}\,\Omega}$$

where $\rho_c = \frac{1}{\sigma}$ = Resistivity of the conductor in Ω-m

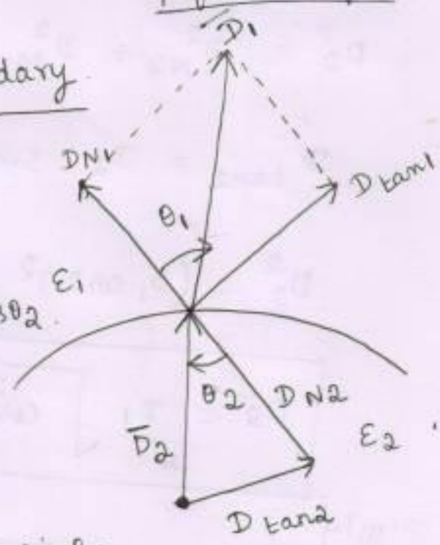
$\sigma \rightarrow$ conductivity of the material (siemens/meter or S/m)

$S \rightarrow$ surface area (m^2).

BOUNDARY CONDITIONS BETWEEN TWO PERFECT DIELECTRIC MATERIALS

Refraction of $\bar{D}$ at the boundary

Refraction of $\bar{D}$



Directions of $\bar{D}$ & $\bar{E}$ change at the boundary between two dielectrics.

$D_{N1} = D_1 \cos\theta_1 \; ; \; D_{N2} = D_2\cos\theta_2$.

$$\boxed{D_{N1} = D_{N2}}$$

$D_1\cos\theta_1 = D_2\cos\theta_2$.

$D_{tan1} = D_1\sin\theta_1 \; ; \; D_{tan2} = D_2\sin\theta_2$.

$$\boxed{\frac{D_{tan1}}{D_{tan2}} = \frac{\varepsilon_1}{\varepsilon_2}}$$

$$\frac{D_1 \sin\theta_1}{D_2 \sin\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{D_{tan1}}{D_{tan2}}$$

$$\tan\theta_1 = \frac{D_{tan1}}{D_{N1}} \quad ; \quad \tan\theta_2 = \frac{D_{tan2}}{D_{N2}}$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{D_{tan1}}{D_{N1}}\left(\frac{D_{N2}}{D_{tan2}}\right)$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{D_{tan1}}{D_{tan2}} = \frac{\varepsilon_1}{\varepsilon_2}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}}$$

This is called law of refraction

$$D_2^2 = D_{N2}^2 + D_{tan}^2 = (D_1 \cos\theta_1)^2 + D_{tan2}^2$$

$$D_{tan2} = D_2 \sin\theta_2 = D_1 \sin\theta_1 \frac{\varepsilon_2}{\varepsilon_1}$$

$$D_2^2 = (D_1 \cos\theta_1)^2 + \left(D_1 \sin\theta_1 \frac{\varepsilon_2}{\varepsilon_1}\right)^2$$

$$\boxed{D_2 = D_1 \sqrt{\cos^2\theta_1 + \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2 \sin^2\theta_1}}$$

III^{ly}

$$E_2 = E_1 \sqrt{\sin^2\theta_1 + \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2 \cos^2\theta_1}$$

$|\bar{D}_1| = |\bar{D}_2|$ if $\theta_1 = \theta_2 = 0°$

$|E_1| = |\bar{E}_2|$ if $\theta_1 = \theta_2 = 90°$.